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A NOTE ON THE FAILURE OF EDUCATED PERSONS TO UNDERSTAND SIMPLE GEOMETRICAL FACTS

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The following was given, as the third in order of a set of eight paragraphs to be read, to twenty-seven adult students in a Summer Session course in Mental Tests.

Read this paragraph. Then read the questions. Answer them, reading the paragraph again as much as is necessary.

Polygon means in geometry, a figure enclosed by any number of lines—the sides—which intersect in pairs at the corners or vertices. If the sides are coplanar, the polygon is said to be "plane"; if not, then it is a "skew" or "gauche" polygon. If the figure lies entirely to one side of each of the bounding lines the figure is "convex"; if not it is "concave." A "regular" polygon has all its sides and angles equal; if the sides and angles be not equal the polygon is "irregular." Of polygons inscriptible in a circle an equilateral figure is necessarily equiangular, but the converse is only true when the number of sides is odd. The term regular polygon is usually restricted to "convex" polygons.

1. (a) What does "coplanar" mean?...................................
   (b) Are all regular polygons convex?...............................
2. (a) Are all polygons either plane or skew?..........................
   (b) Are all polygons either convex or concave?.....................
3. (a) Are all equilateral polygons inscribed in a circle regular?........
   (b) Are all regular polygons equiangular?..........................
4. Draw an irregular concave polygon that is a quadrilateral with sides approximately 2, 2, 1½ and 1½ inches long.
5. What do we commonly call a regular four-sided polygon?............
6. (a) What figure would be made by placing six regular three-sided polygons of equal area so that six vertices, one from each polygon, met in a common point and so that there was no overlap of area? ....
   (b) What figure would be approximated by a regular polygon of n sides as n is increased?...................................

Sixty minutes were allowed in all, and such directions were given as to make it probable that from six to ten minutes were spent on this paragraph. It is highly probable that nine out of ten of the individuals tested had studied geometry for a year or more. It is also highly probable that they were above the average in their high school classes in mathematical ability, general scholarship, and interest in geometry. Such being the case, the frequency, variety, and gravity of their errors in questions 1, 4, 5, and 6 of this task* seem instructive to teachers of mathematics, though just what they should teach us is not so certain.

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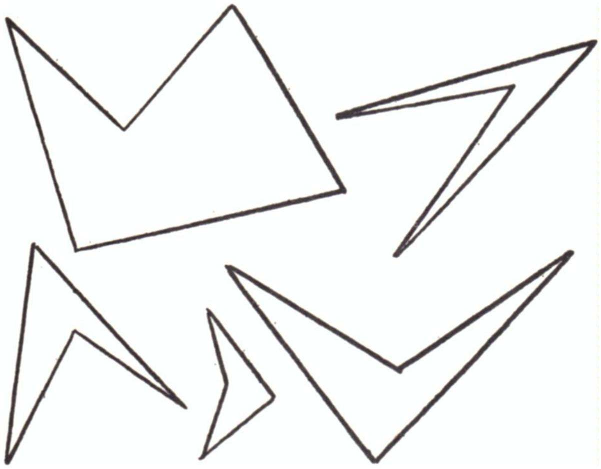
* The questions of 2 were answered correctly by 20 of the 27. The questions of 3 were answered correctly by 21 of the 27. Nothing will be said further about them.

Eight, or less than one in three, answered 1*a* correctly. Seven omitted it. The other twelve gave "plane," "flat," "plane polygon," "polygon with plane sides," "sides of a plane polygon," "parallel," "regular," "at right angles," "sides intersect in pairs at corners," "regular, opposite parallel," and "together as a plane." It seems that a clear notion of a plane in condition for ready use as an instrument of thought was absent from the minds of over a third of this group. Any working mastery of "co" was probably as rare, or rarer.

Number 4 was omitted by eleven and drawn as convex* by eleven. Of the five who drew a concave polygon, one used five sides and two made the sides far shorter than the specifications, as shown in figure 1.



It appears that less than one in five of the group had the ability to read and apply quickly the perfectly clear definition of concave polygon given in the paragraph.

Number 5 was answered correctly by thirteen of the group and half correctly by one ("square or rectangle"). The others gave such answers as "quadrilateral," "cube," "trapezoid," "parallelogram," "convex," "rectangle," "oblong," and "opposite sides equal." It thus appears that only half of the group have been equipped by their study of geometry with the ability to understand and apply readily, "A regular polygon has all its sides and angles equal"; and this would seem to imply that their actual working knowledge of line, angle, side, and polygon was very weak. At least they did not in the test do much that was useful with them.

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* Rectangles, trapezoids, a hexagon and a cube were among these.

Number 6*a* was answered correctly* by fourteen. Two others showed some geometrical appreciation by answering "pyramid." The remaining eleven either omitted 6*a* or gave "rectangle," "circle," "prism," or "equilateral triangle."

Number 6*b* was answered correctly by twelve. Thirteen omitted it. Two wrote "rectangle."

Considering 1*a*, 4, 5, 6*a* and 6*b* as a total, six individuals had none right, seven had one right, five had two right, five had three right, two had four right, and two had five right. Thus it seems conservative to say that two out of three lacked an adequate, fluent, *working* knowledge of what a "point," a "line," an "angle," a "polygon," a "quadrilateral," "approximate," and "*n* sides" mean. They could not use these meanings effectively in slightly new and complex problems. The conclusion seems unavoidable that the common expectations from the study of geometry are pitched far too high. Whether this is due to inadequate teaching or to the essential difficulty of the concepts and terms and relations involved, I shall not try to decide.

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* Not fully, however. Only one of the entire group noted that the figure would be a regular hexagon.